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Bianchi Type V Cosmological Models with Constant Deceleration Parameter in General Relativity

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Abstract We investigate Bianchi type V cosmological models with bulk viscous fluid source. Exact solutions of the Einstein field equations are presented via a suitable power law assumption for the Hubble parameter. We show that the corresponding solutions retain the well established features of the standard cosmology and in addition, are in accordance with recent type Ia supernovae observations. Some observational parameters for the models have also been discussed.

Keywords Hubble's parameter · Deceleration parameter · Bianchi space-time · Cosmological models · Anisotropy · Bulk viscosity

1 Introduction

Einstein's general theory of relativity has been successfully used as the theory governing the large-scale structure of reasonable models of the physical universe. Within this framework, spatially homogeneous, isotropic universe models (the Friedmann-Robertson-Walker or FRW universes) provide common meeting ground for an enormous variety of observational data. By discussing these data in the context of FRW cosmological model, one can appreciate their cosmological relevance. The uncertainty that remains concerns the early thermodynamic evolution of such a universe which affects the detailed dynamic behaviour. These space-times are good models for time after the era in which the universe became transparent to radiation and their extrapolation to earlier times is totally ungrounded. Because of their very high symmetry they could be poor approximations at early times. Consequently it is necessary to investigate more realistic universe models.

There are theoretical arguments [9, 30] and recent experimental data of the cosmic microwave background radiation anisotropies which support the existence of an anisotropic

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phase that approaches an isotropic one [22]. Besides, if we intend to avoid the assumptions of special initial conditions tacitly implied in FRW cosmologies, we should study more appropriate models in which anisotropies, supposed to be damped out in the course of evolution [15, 16], can exist from the very beginning. Therefore it makes sense to investigate spatially homogeneous and anisotropic cosmological models which provide richer structure both geometrically and physically than FRW models and play significant role in the description of early universe.

There has been considerable interest in the study of spatially homogeneous, anisotropic cosmological models. Bianchi type V space-times, being a straight forward generalization of FRW universe with negative curvature are interesting to study because they contain isotropic special cases and allow arbitrarily small anisotropy levels at any instant of cosmic time. Farnsworth [12], Collins [11], Maartens and Nel [27], Wainwright et al. [63], have studied Bianchi type V cosmological models. Nayak and Sahoo [34] have investigated Bianchi type V models for matter distribution admitting anisotropic pressure and heat flow. Ram [42] has obtained exact solution of Bianchi type V space-time for perfect fluid distribution. Roy and Singh [48, 49] have investigated Bianchi type V models with electromagnetic field. Banerjee and Sanyal [6] have studied Bianchi type V cosmological models with viscosity and heat flow. Coley [10] has investigated Bianchi type V imperfect fluid cosmological models. Roy and Prasad [47] have studied LRS Bianchi type V cosmological models of local embedding class one containing perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have also been investigated by Bali and Yadav [4], Bali and Singh [3], Bali and Meena [2], Bali and Jain [1], Singh and Agrawal [56], Lorenz [26], Singh and Chaubey [57, 58], Pradhan and Yadav [40], Singh [54, 55].

The investigation of relativistic cosmological models usually has the cosmic fluid as perfect fluid. However, these models do not incorporate dissipative mechanisms responsible for smoothing out initial anisotropies. It is believed that during neutrinos decoupling, the matter behaved like a viscous fluid in the early stages of evolution. Misner [30, 31] suggested that strong dissipative mechanism due to neutrino viscosity considerably reduce the anisotropy of the black body radiation. Viscosity mechanism in cosmology can explain anomalously high entropy per baryon in the present universe [61, 62].The role of bulk viscosity in the cosmic evolution, especially at its early stages seems to be significant. Bulk viscosity associated with the grand-unified-theory phase transition [23] may lead to an inflationary scenario [13, 35, 60]. Cosmological models with bulk viscosity have been discussed by a number of authors in various contexts [5, 19, 32, 36, 51].

In this paper, we investigate the evolution of Bianchi type V cosmological models with bulk viscous fluid in general relativity. We obtain exact solutions of Einstein field equations assuming a law of variation for Hubble's parameter in Bianchi type V space-time which yields a constant value of deceleration parameter. The law of variation explicitly determines the scale factors. Physical behaviour of the models have been studied. The observational parameters such as look-back time, proper distance, luminosity distance and angular diameter distance for the models have also been discussed.

2 Metric and Field Equations

We consider the homogeneous and anisotropic Bianchi type V space-time described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\left\{B^2(t)dy^2 + C^2(t)dz^2\right\}. \tag{1}$$

Cosmic matter is assumed to consist of bulk viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p} g_{ij}, \tag{2}$$

where ρ, $\bar{p}$ are respectively the energy density, dissipative pressure and v_i is the four-velocity vector of the fluid satisfying $v_i v^i = -1$. The dissipative pressure $\bar{p}$ is related to the hydrostatic pressure p by $\bar{p} = p - \zeta v^i_{;i}$, where ζ is coefficient of bulk viscosity that determines the magnitude of viscous stress relative to expansion. We shall use non-causal theory to study the dissipative mechanism. On thermodynamical grounds bulk viscosity coefficient ζ is positive, assuming that the viscosity pushes the dissipative pressure towards negative values.

The Einstein field equations

$$R^j_i - \frac{1}{2} R g^j_i = -8\pi G T^j_i + \Lambda g^j_i, \tag{3}$$

in case of a Bianchi type V universe for bulk viscous fluid distribution in comoving coordinates lead to

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{4}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \tag{5}$$

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB}, \tag{6}$$

$$8\pi G\rho + \Lambda = -\frac{3\alpha^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}, \tag{7}$$

$$0 = \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}. \tag{8}$$

In the above and elsewhere overhead dot stands for ordinary time-derivative of the concerned quantity.

We define the average scale factor R as $R^3 = ABC$. Integrating (8), we obtain

$$A^2 = BC. \tag{9}$$

From (4), (5) and (6), we get

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R}, \tag{10}$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{k_1}{R^3}, \tag{11}$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{k_1}{R^3}, \tag{12}$$

where k_1 is constant of integration.

Equations (10)–(12) on integration give

$$A = m_1 R, \tag{13}$$

$$B = m_2 R \exp\left(-k_1 \int \frac{dt}{R^3}\right), \tag{14}$$

$$C = m_3 R \exp\left(k_1 \int \frac{dt}{R^3}\right), \tag{15}$$

where m_1, m_2 and m_3 are constants of integration satisfying $m_1 m_2 m_3 = 1$. In view of (9), we get $m_1 = 1$ and $m_2 = m_3^{-1}$.

In analogy with FRW universe, we define a generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{16}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\frac{\dot{H}}{H^2} - 1, \tag{17}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors in the x, y and z directions respectively. The anisotropy parameter $\bar{A}$ is defined as

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{18}$$

We introduce volume expansion θ and shear scalar σ as usual

$$\theta = v^i_{;i}, \qquad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij},$$

σ_{ij} being shear tensor. In the above the semicolon stands for covariant differentiation.

For the Bianchi type V metric expressions for θ and σ come out to be

$$\theta = \frac{3\dot{R}}{R}, \tag{19}$$

$$\sigma = \frac{k_1}{R^3}. \tag{20}$$

Equations (4)–(7) can be recast in terms of H, σ and q as

$$8\pi G\bar{p} - \Lambda = \frac{\alpha^2}{R^2} + H^2(2q - 1) - \sigma^2, \tag{21}$$

$$8\pi G\rho + \Lambda = -\frac{3\alpha^2}{R^2} + 3H^2 - \sigma^2. \tag{22}$$

From (22) we observe that

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{3\alpha^2}{R^2\theta^2} - \frac{\Lambda}{\theta^2}. \tag{23}$$

Therefore $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Thus a positive Λ restricts the upper limit of anisotropy whereas a negative Λ will increase the anisotropy. From (21) and (22), we obtain

$$\frac{d\theta}{dt} = -4\pi G(\rho + 3\bar{p}) - 2\sigma^2 - \frac{\theta^2}{3} + \Lambda, \tag{24}$$

which is the Raychaudhuri equation for the given distribution.

Therefore for $\Lambda \le 0$ and $\zeta = 0$, the universe will always be in decreasing phase provided the strong energy condition holds [14]. In this case we have

$$\frac{d\theta}{dt} \le -\frac{1}{3}\theta^2, \tag{25}$$

which integrates to give

$$\frac{1}{\theta} \ge \frac{1}{\theta_0} + \frac{t}{3}, \tag{26}$$

where θ_0 is the initial value of θ. If $\theta_0 < 0$ initially, θ will diverge ($\theta \to -\infty$) for $t \le \frac{3}{|\theta_0|}$. A positive Λ will arrest the rate of decrease. Also the presence of viscosity will slow down the rate of decrease. It is to note that the rate of decrease of volume expansion is faster in anisotropic background in comparison to that in an isotropic one. Also $\dot{\sigma} = -3\sigma H$ implying σ decreases in an evolving universe and for infinitely large value of R, σ becomes negligible. If we restrict ourselves to dust only, then from (21) and (22), we obtain

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{\alpha^2}{R^2} + \frac{k_1^2}{R^6} - \Lambda - 8\pi G\zeta\theta = 0, \tag{27}$$

$$\frac{3\dot{R}^2}{R^2} = 8\pi G\rho + \frac{3\alpha^2}{R^2} + \frac{k_1^2}{R^6} + \Lambda. \tag{28}$$

We observe that for $\Lambda \ge 0$, each term on the right hand side of (28) is non-negative. Thus $\dot{R}$ does not change sign and we get ever-expanding models. For $\Lambda < 0$, however, we can get universe that expand and then recontract. Also for $\Lambda \ge 0$, $\rho < \rho_c$ where $\rho_c = \frac{3H^2}{8\pi G}$ is the critical density.

It is to note that energy density of all known forms of ponderable matter is positive. At the initial epoch of the evolution when the average scale factor was close to zero, the energy density of the universe is believed to be infinitely large. During expansion phase of the universe i.e. with increase of R, the energy density decreases and an infinitely large R corresponds to a ρ close to zero. From (22), we can draw conclusions similar to Saha [50] in Bianchi type V space-time also.

3 Universe with Constant Deceleration Parameter

Among the physical quantities of interest in cosmology the deceleration parameter q is currently a serious candidate to describe the dynamics of the universe. The prediction of standard cosmology, that the universe at present is decelerating is contradictory to the recent observational evidences of the high redshift of type Ia supernovae [17, 20, 37, 45, 46, 59]. Observations reveal that instead of slowing down, the expanding universe is speeding up. Models with constant deceleration parameter have received considerable attention recently. Initially Berman [8] proposed a variation law for Hubble's parameter for spatially homogeneous and isotropic FRW metric that yields a constant value of deceleration parameter. The variation of Hubble's parameter as assumed is consistent with observations. Cosmological models with constant deceleration parameter have been studied by

Berman and Gomide [7], Maharaj and Naidoo [29], Johri and Desikan [18], Singh and Desikan [53], Pradhan and Vishwakarma [39], Pradhan and Aotemshi [38], Singh and Kumar [52], Kumar and Singh [21], Rahman et al. [41] and Reddy et al. [43, 44] to name only a few.

We intend to extend the results of Berman [8] and Berman and Gomide [7] to Bianchi type V space-time to solve Einstein's field equations. We formulate a similar type of law for variation of the Hubble's parameter that yields a constant value of deceleration parameter. We assume the variation of the Hubble's parameter given by

$$H = kR^{-m}, \tag{29}$$

where $k > 0$ and $m \geq 0$ are constants. For this choice of Hubble's parameter, the deceleration parameter q comes out to be constant, i.e.

$$q = m - 1. \tag{30}$$

We observe that for $m > 1$, the model represents a decelerating universe and $m < 1$ corresponds to accelerating phase of the universe. When $m = 1$, we obtain $H = \frac{1}{t}$ and $q = 0$. Since the deceleration parameter q is zero in the model, every galaxy moves with constant speed. Therefore for $m = 1$, we recover anisotropic Milne model [25]. For $m = 0$, we get $H = k$ (constant) and $q = -1$. We observe that the Hubble parameter being a large-scale property of the universe is constant in time. Therefore the observed Hubble parameter is a true constant equal to its present value H_0. For this value of m, the model may be considered as a steady-state model of the universe. In this case decaying energy densities $\frac{3\alpha^2}{R^2}$ and σ^2 associated with curvature and anisotropy of the open anisotropic space-time seem to behave like a creation field [24, 33, 62]. Also deceleration parameter q for this model is negative. Therefore it represents accelerating phase of the universe.

Integrating (29), we obtain

$$R = (mkt + t_1)^{\frac{1}{m}}, \quad \text{for } m \neq 0 \tag{31}$$

and

$$R = \exp\{k(t - t_0)\}, \quad \text{for } m = 0, \tag{32}$$

where t_1 and t_0 are constants of integration.

Using (31) in (13)–(15), we obtain

$$A = (mkt + t_1)^{\frac{1}{m}}, \tag{33}$$

$$B = m_2(mkt + t_1)^{\frac{1}{m}} \exp\left\{\frac{-k_1}{k(m-3)}(mkt + t_1)^{\frac{m-3}{m}}\right\}, \tag{34}$$

$$C = m_2^{-1}(mkt + t_1)^{\frac{1}{m}} \exp\left\{\frac{k_1}{k(m-3)}(mkt + t_1)^{\frac{m-3}{m}}\right\}. \tag{35}$$

For this solution the metric (1) assumes the following form after suitable transformation of coordinates

$$ds^2 = -dT^2 + (mkT)^{\frac{2}{m}}\left[dX^2 + \exp\left\{2\alpha X - \frac{2k_1}{k(m-3)}(mkT)^{\frac{m-3}{m}}\right\}dY^2\right]$$
$$+ (mkT)^{\frac{2}{m}}\exp\left\{2\alpha X + \frac{2k_1}{k(m-3)}(mkT)^{\frac{m-3}{m}}\right\}dZ^2. \tag{36}$$

Equations (13)–(15) with the help of (32) give rise to

$$A = \exp\{k(t-t_0)\}, \tag{37}$$

$$B = m_2 \exp\left\{k(t-t_0) + \frac{k_1}{3k}e^{-3k(t-t_0)}\right\}, \tag{38}$$

$$C = m_2^{-1}\exp\left\{k(t-t_0) - \frac{k_1}{3k}e^{-3k(t-t_0)}\right\}. \tag{39}$$

The metric (1) for this solution reduces to

$$ds^2 = -dT^2 + e^{2kT}\left\{dX^2 + \exp\left(2\alpha X + \frac{2k_1}{3k}e^{-3kT}\right)dY^2\right\}$$
$$+ e^{2kT}\left\{\exp\left(2\alpha X - \frac{2k_1}{3k}e^{-3kT}\right)\right\}dZ^2. \tag{40}$$

4 Analysis and Discussion

For the model (36) pressure p and energy density ρ are given by

$$8\pi Gp = \frac{2m-3}{m^2T^2} + \frac{\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{k_1^2}{(mkT)^{\frac{6}{m}}} + \frac{24\pi G\zeta}{mT} + \Lambda, \tag{41}$$

$$8\pi G\rho = \frac{3}{m^2T^2} - \frac{3\alpha^2}{(mkT)^{\frac{2}{m}}} - \frac{k_1^2}{(mkT)^{\frac{6}{m}}} - \Lambda. \tag{42}$$

Expansion scalar θ, shear σ and anisotropy parameter $\bar{A}$ can be expressed as

$$\theta = \frac{3}{mT}, \tag{43}$$

$$\sigma = \frac{k_1}{(mkT)^{\frac{3}{m}}}, \tag{44}$$

$$\bar{A} = \frac{2k_1^2}{3k^2}(mkT)^{\frac{2(m-3)}{m}}. \tag{45}$$

This model is valid for $m \neq 0$ and $m \neq 3$. The coefficient of bulk viscosity is assumed to be a simple power function of energy density ρ [28]:

$$\zeta = \zeta_0 \rho^{\beta}, \tag{46}$$

where ζ_0 (≥ 0) and β (≥ 0) are constants. If $\beta = 1$, we obtain radiating fluid whereas $\beta = 3/2$ may correspond to string dominated universe [32]. However more realistic models are based on β lying in the regime $0 \leq \beta \leq 1/2$ [51]. The model has point singularity at

$T = 0$ for $m > 3$. We observe that the model starts with ρ, p, ζ, θ, σ all infinite and evolves to isotropy with $p \to \Lambda$, $\rho \to -\Lambda$ and $\zeta \to$ constant as $T \to \infty$ provided $m < 3$. The anisotropy parameter $\bar{A}$ vanishes for large values of T. Therefore at late times, the model represents an isotropic universe dominated by vacuum energy. For the model (36), Hubble parameter H is given by

$$H = \frac{1}{mT} = \frac{1}{(q+1)T}. \tag{47}$$

Thus $T = \frac{H^{-1}}{q+1}$. We observe that the age of the universe increases as q decreases, being H^{-1} for $q = 0$. Physical components of the conformal curvature tensor are given by

$$C_{(1212)} = \frac{1}{3}\frac{k_1^2}{(mkT)^{\frac{6}{m}}} - \frac{kk_1}{(mkT)^{\frac{m+3}{m}}}, \tag{48}$$

$$C_{(1313)} = \frac{1}{3}\frac{k_1^2}{(mkT)^{\frac{6}{m}}} + \frac{kk_1}{(mkT)^{\frac{m+3}{m}}}, \tag{49}$$

$$C_{(2323)} = -\frac{2}{3}\frac{k_1^2}{(mkT)^{\frac{6}{m}}}, \tag{50}$$

$$C_{(1224)} = -C_{(1334)} = \frac{\alpha k_1}{(mkT)^{\frac{5}{m}}}. \tag{51}$$

We observe that $C_{(hijk)} \to 0$ for $T \to \infty$. Thus the model represents a shearing, non-rotating and expanding model of the universe with a big bang start approaching isotropy at late times. For large values of T, the model becomes conformally flat. The integral

$$\int_{T_0}^{T} R^{-1}(T')dT' = \frac{1}{k(m-1)}\left[(mkT')^{\frac{m-1}{m}}\right]_{T_0}^{T} \tag{52}$$

is finite provided $m \neq 1$. Therefore particle horizon exists in the model. It is to note that for $m = 1$, the model does not have horizon.

We investigate the consistency of our model (36) with the observational parameters. We measure the physical parameters such as redshift, look-back time, proper distance, luminosity distance, angular diameter etc.

Look-Back Time

The look-back time T_L is defined as the elapsed time between the present age of universe T_0 and the time T when the light from a cosmic source at a particular redshift z was emitted. In the context of our model it is given by

$$T_L = T_0 - T = \int_{R}^{R_0} \frac{dR}{\dot{R}}, \tag{53}$$

where R_0 is the present day scale factor of the universe and

$$\frac{R_0}{R} = 1 + z. \tag{54}$$

For the model (36), we have

$$\frac{R_0}{R} = 1 + z = \left(\frac{T_0}{T}\right)^{\frac{1}{m}} \tag{55}$$

implying

$$T = (1+z)^{-m} T_0. \tag{56}$$

Using (43), we obtain

$$T_0 - T = \frac{1}{m H_0}\{1 - (1+z)^{-m}\}. \tag{57}$$

Thus

$$H_0(T_0 - T) = \frac{1}{m}\{1 - (1+z)^{-m}\}. \tag{58}$$

For small z, we get

$$\begin{aligned} H_0(T_0 - T) &= z - \frac{(m+1)z^2}{2!} + \frac{(m+1)(m+2)z^3}{3!} - \cdots \\ &= z - \left(1 + \frac{q}{2}\right) z^2 + \cdots . \end{aligned} \tag{59}$$

If we take $z \to \infty$, (56) and (58) give $H_0 T_0 = \frac{1}{m}$ and for $m = \frac{3}{2}$, we obtain the well-known Einstein-de Sitter result

$$H_0(T_0 - T) = \frac{2}{3}\{1 - (1+z)^{-\frac{3}{2}}\}. \tag{60}$$

Proper Distance

The proper distance $d(z)$ is defined as the distance between a cosmic source emitting light at any instant $T = T_1$ located at $r = r_1$ with redshift z and an observer at $r = 0$ and $T = T_0$ receiving the light from the source emitted i.e.

$$d(z) = r_1 R_0, \tag{61}$$

where

$$r_1 = \int_{T_1}^{T_0} \frac{dT}{R(T)} = \frac{R_0^{-1} H_0^{-1}}{m-1}\{1 - (1+z)^{1-m}\}, \quad m \neq 1. \tag{62}$$

Hence

$$d(z) = r_1 R_0 = \frac{H_0^{-1}}{m-1}\{1 - (1+z)^{1-m}\}. \tag{63}$$

For small z, we obtain

$$H_0 d(z) = z - \frac{m}{2} z^2 + \cdots = z - \frac{1}{2}(q+1) z^2 + \cdots . \tag{64}$$

If $m > 1$, at T_0 the observer can not see any source beyond the proper distance of the particle horizon. Therefore, for $m > 1$

$$d(z = \infty) = \frac{H_0^{-1}}{m-1} \tag{65}$$

and for $m < 1$, $d(z = \infty)$ is always infinite.

Luminosity Distance

We may define the luminosity distance d_L of a light source as

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}} = r_1 R_0 (1+z), \tag{66}$$

where L is the absolute luminosity and l is the apparent luminosity of source. From (61) and (66), we get

$$d_L = d(z)(1+z) \tag{67}$$

which together with (63) gives

$$d_L = \frac{H_0^{-1}}{m-1}\{(1+z) - (1+z)^{2-m}\}. \tag{68}$$

For $m = \frac{3}{2}$, we obtain

$$d_L = 2H_0^{-1}\{(1+z) - (1+z)^{\frac{1}{2}}\}. \tag{69}$$

For small z, (68) gives

$$H_0 d_L = z + \frac{(2-m)z^2}{2} + \cdots = z + \frac{(1-q)z^2}{2} + \cdots. \tag{70}$$

We observe that for the same redshift the luminosity distance is larger for smaller values of q. For $q = 1$, we obtain

$$d_L = \frac{z}{H_0} \tag{71}$$

and for $q = 0$,

$$d_L = z + \frac{z^2}{2}. \tag{72}$$

Angular Diameter

The angular diameter of a light source of diameter D at $r = r_1$ and $T = T_1$ observed at $r = 0$ and $T = T_0$ is given by

$$\delta = \frac{D}{r_1 R(T_1)} = \frac{D(1+z)^2}{d_L}. \tag{73}$$

The angular diameter distance d_A is defined as the ratio of the source diameter to its angular diameter

$$d_A = \frac{D}{\delta} = r_1 R(T_1) = d_L (1+z)^{-2}. \tag{74}$$

Using (68), we get

$$d_A = \frac{H_0^{-1}}{m-1}\left\{\frac{1-(1+z)^{1-m}}{1+z}\right\}. \tag{75}$$

The maximum value of d_A in case of the model (36) occurs at

$$1 + z_M = (m)^{\frac{1}{m-1}}. \tag{76}$$

For $m = \frac{3}{2}$, we obtain Einstein-de Sitter result i.e. $z_M = \frac{5}{4}$.

We now discuss the model (40). Pressure p and energy density ρ for the model are given by

$$8\pi G p = \alpha^2 e^{-2kT} - k_1^2 e^{-6kT} - 3k^2 + 24\pi G k\zeta + \Lambda, \tag{77}$$

$$8\pi G\rho = -3\alpha^2 e^{-2kT} - k_1^2 e^{-6kT} + 3k^2 - \Lambda. \tag{78}$$

Expressions for expansion scaler θ, shear σ and anisotropy parameter $\bar{A}$ are

$$\theta = 3k, \tag{79}$$

$$\sigma = k_1 e^{-3kT}, \tag{80}$$

$$\bar{A} = \frac{k_1}{6k^2} e^{-6kT}. \tag{81}$$

Physical components of conformal curvature tensor are given by

$$C_{(1212)} = \frac{1}{3} k_1^2 e^{-6kT} - k k_1 e^{-3kT}, \tag{82}$$

$$C_{(1313)} = \frac{1}{3} k_1^2 e^{-6kT} + k k_1 e^{-3kT}, \tag{83}$$

$$C_{(2323)} = -\frac{2}{3} k_1^2 e^{-6kT}, \tag{84}$$

$$C_{(1224)} = -C_{(1334)} = \alpha k_1 e^{-5kT}. \tag{85}$$

We observe that the model has no initial singularity. It starts evolving at $T = 0$ with ρ, p, θ, σ and $\bar{A}$ are all finite. The expansion scalar θ is constant throughout the evolution of the universe and therefore the model represents uniform expansion. As $T \to \infty$, pressure and density become constants whereas anisotropy parameter and shear scalar become zero. Therefore the model approaches isotropy for large values of T. There exists event horizon in the model. Also $C_{(hijk)} \to 0$ as $T \to \infty$ implying that the universe becomes conformally flat for large T. We observe that for this model Hubble parameter, being a large scale property of the universe is constant in time. Therefore it may be assumed to represent a steady-state model of the universe with curvature energy density $3\alpha^2 e^{-2kT}$ and anisotropy energy density $k_1^2 e^{-6kT}$ appear to behave like a creation field. Over the last few years there are growing evidences in favour of the current accelerated expansion of our universe. Recent observations of type Ia supernovae [17, 20, 37, 45, 46, 59] strongly suggest this acceleration. For the model (40), $m = 0$ and we get $q = -1$. Thus the model represents an accelerating universe. Therefore the model is consistent with the cosmological observations.

To investigate the consistency of the model (40), we measure the physical parameters such as redshift, look-back time, proper distance, luminosity distance, angular diameter etc. as we have obtained for the model (36).

Look-back Time:

$$H_0(T_0 - T) = \ln(1+z). \tag{86}$$

For small z, we have

$$H_0(T_0 - T) \approx z. \tag{87}$$

Proper Distance:

$$d(z) = \frac{z}{H_0}. \tag{88}$$

Thus proper distance $d(z)$ is linear with redshift z.

Luminosity Distance:

$$d_L = d(z)(1+z) = \frac{z(1+z)}{H_0}. \tag{89}$$

Angular Diameter Distance:

$$d_A = \frac{z}{H_0(1+z)}. \tag{90}$$

Thus d_A approaches the finite constant H_0^{-1} as $z \to \infty$. Hence objects with large redshift look very faint.

5 Conclusion

The proposal of a law of variation of Hubble' parameter that yields a constant value of deceleration parameter is discussed in anisotropic Bianchi type V space-time in general relativity. This law, together with Einstein's field equations leads to a number of new solutions of Bianchi type V space-time. We have obtained two physically viable models of the universe. For $m \neq 0$, the model evolves with a big bang start at $T = 0$ and approaches isotropy as $T \to \infty$ provided $m < 3$. For large T, pressure and energy density become constant such that $p = -\rho$ implying that at late times the universe is dominated by vacuum energy. The rate of expansion in the model slows down tending to zero as $T \to \infty$. For $m = 0$, the model has no singularity. It starts with finite values of kinematical and geometrical parameters at $T = 0$ and expands uniformly. For this model $q = -1$ which is compatible with the recent supernovae Ia observations that the universe is undergoing a late time acceleration. Hubble parameter H which is a large scale property of the universe is constant in time for this model. Therefore, we may consider it to represent a steady-state model of the universe. It is to note that the introduction of bulk viscosity does not automatically exclude the appearance of big bang type singularity.

We observe that for a decelerating universe, we require $m > 1$ and for $0 \leq m < 1$, we are left with an accelerated universe. When $m = 1$, we obtain $H = \frac{1}{T}$ and $q = 0$. Since the deceleration parameter q is zero in the model, every galaxy moves with constant speed. Therefore for $m = 1$, we recover an anisotropic Milne model. Both the models approach isotropy for large values of T and they become conformally flat at late times. We have also taken an account of the consistency of our models with observational parameters such as look-back time, proper distance, luminosity distance and angular diameter. In summary, we have extended the law of variation of Hubble parameter proposed by Berman [8] to Bianchi type V space-time to investigate the exact solutions of Einstein's field equations. The solutions obtained are consistent with the results of recent observations.

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